

Pythagorean Combinations for LEGO Robot Building

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LEGO Units

One LEGO unit = 8mm (between centers of adjacent liftarm holes)

Standard Botball kit gears:

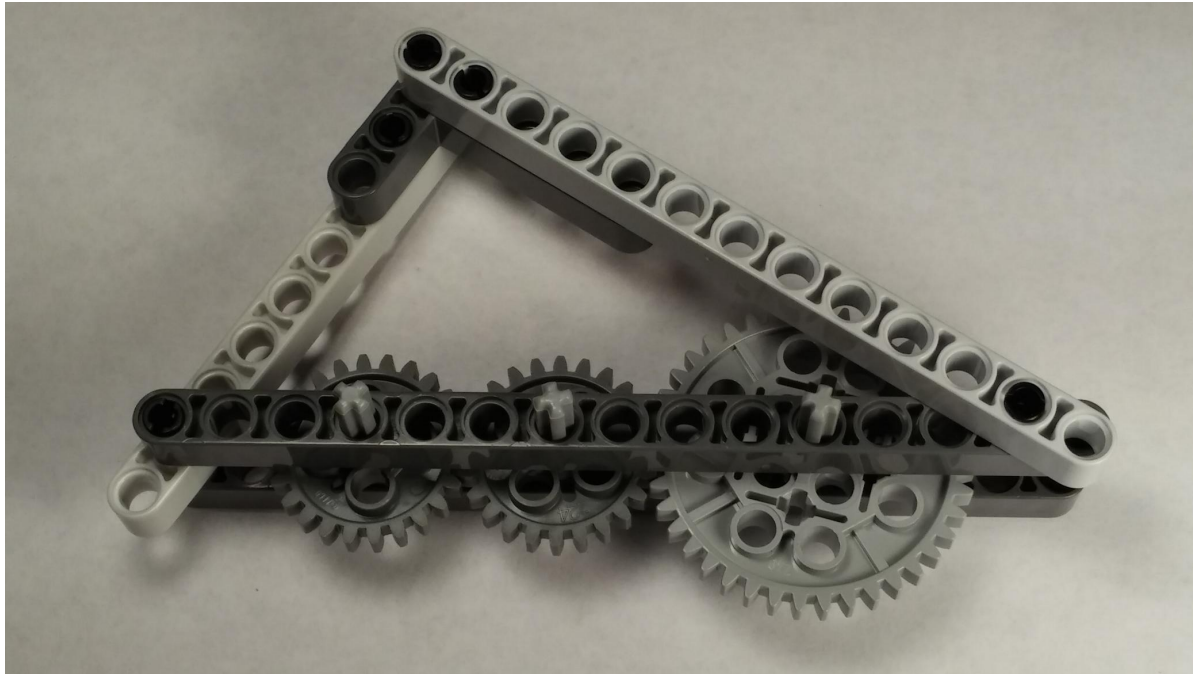
Description	Part #	Radius in LEGO Units	Quantity
8 Tooth	3647	0.5	12
16 Tooth	4019	1	6
24 Tooth	3648	1.5	8
40 Tooth	3649	2.5	6

(Radius is half the center-to-center space when two gears are tightly meshed.)

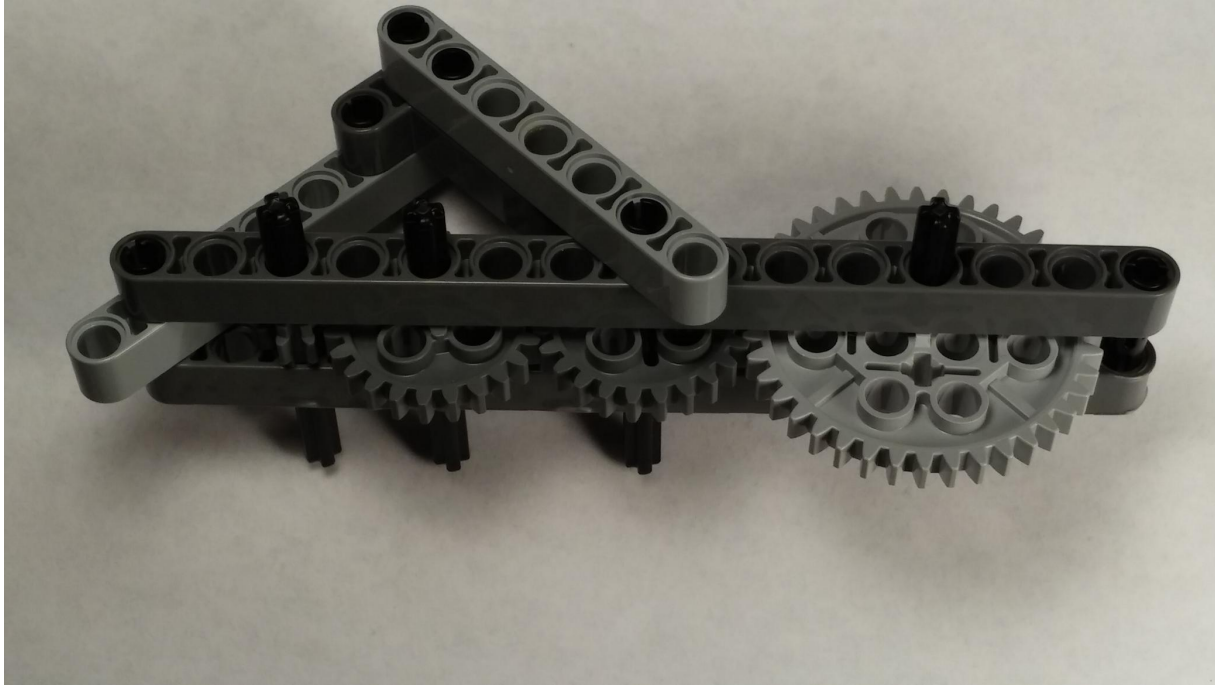
Gears Aligned Along a Liftarm

- 16 Tooth gears can be aligned at distance 2.
- Other gears align at distances of 1, 2, 3, 4, or 5.
- For diagonal placement, can use the Pythagorean Theorem ($a^2+b^2=c^2$), but there are few exact combinations with short side lengths: 3-4-5, 6-8-10, 5-12-13.
- To find combinations that are close enough to satisfying Pythagorean Theorem, the paper provides a macro to generate an Excel spreadsheet of 68 combinations that one can sort by side/hypotenuse lengths, error, or slope (defined here as long leg divided by short leg). Just about all are feasible, and 11 have an (absolute) error of less than 0.1 LEGO units in the hypotenuse.

Example: Approx. Triple with Least Absolute Error (7-11-13)

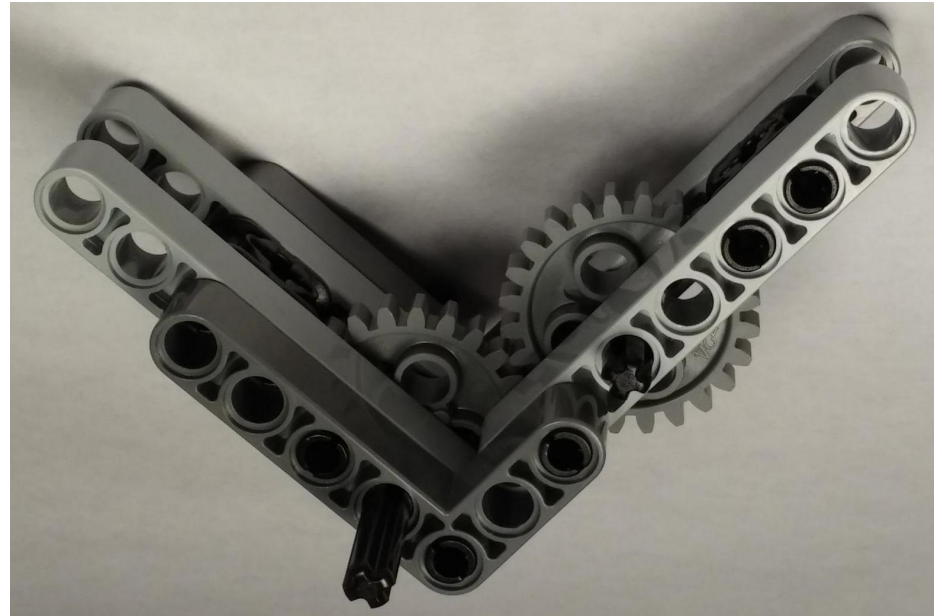
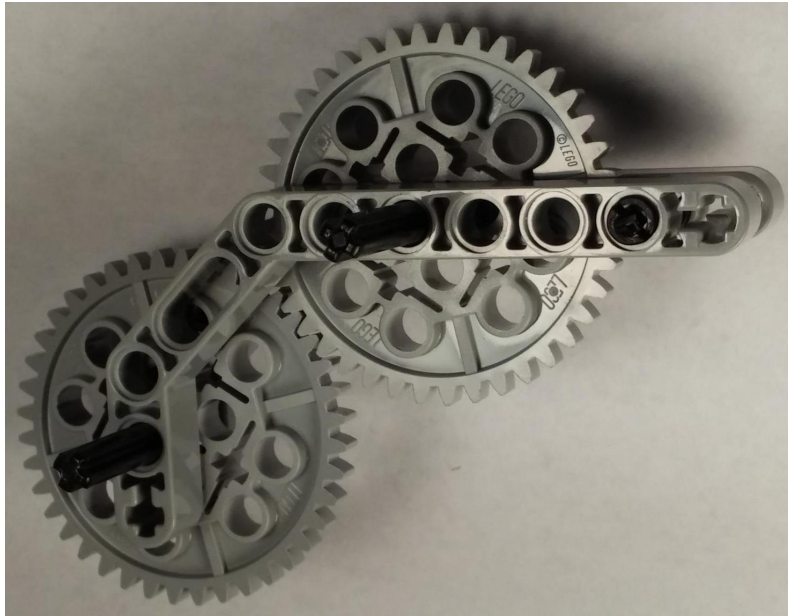


Example: Least Absolute Error for Slope 1 (5-5-7)



Example: Hypotenuse with Gears Only

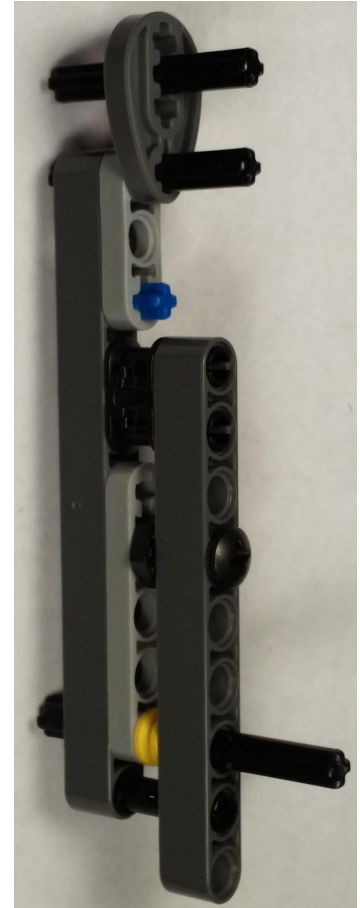
Exact 3-4-5 and largest feasible absolute error (1-3-3)



Half-Unit Spacing

Various parts can be used to place pieces at half-unit spacing within the implicit underlying 3D LEGO grid:

Description	Part #	Quantity
Bush 1/2	4265c	42
Nut 8-32 Keeps (black)		85
1 x 3 Liftarm Thin	6632	16
Triangle	2905	4
Cam	6575	4



Triangles with Half-Unit Sides

- Can modify macro to generate spreadsheet entries including half-unit sides.
- Can place gears along a half-unit side by using 16-Tooth with others at spacing of 1.5, 2.5, or 3.5, or new trick with double bevel gears at spacings of 1.5, 2, 2.5, 3, 3.5, or 4.5:

Description	Part #	Radius in LEGO Units	Quantity
12 Tooth Double Bevel	32270	0.75	4
20 Tooth Double Bevel	32269	1.75	8
36 Tooth Double Bevel	x403	2.25	6

Quarter-Unit Spacing

- Generally harder to achieve, but can at least pair a traditional gear with a double bevel gear along a hypotenuse at spacings of 1.25, 1.75, 2.25, 2.75, 3.25, 3.75, or 4.75.
- A further tweak of the Excel macro can generate approximate Pythagorean triples with such a hypotenuse.

Conclusion

Other odd spacings may also be achievable by interposing metal pieces, bricks, or plates/tiles between liftarms, but we have shown a systematic approach to employ strong diagonal structures while using primarily pieces that are plentiful in the Botball kit and keeping liftarms, gear centers, etc. on a standard grid or half-unit grid.