Pythagorean Approximations for LEGO: Merging Educational Robot Construction with Programming and Data Analysis

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LEGO Units

One LEGO unit = 8mm (between centers of adjacent liftarm holes)

Standard Botball kit gears:

Description	Part #	Radius in LEGO Units	Quantity
8 Tooth	3647	0.5	12
16 Tooth	4019	1	6
24 Tooth	3648	1.5	8
40 Tooth	3649	2.5	6

(Radius is half the center-to-center space when two gears are tightly meshed.)

Gears Aligned Along a Liftarm

- 16 Tooth gears can be aligned at distance 2.
- Other gears align at distances of 1, 2, 3, 4, or 5.
- For diagonal placement, just a few exact combinations with short side lengths from Pythagorean Theorem (a^2+b^2=c^2): 3-4-5, 6-8-10, 5-12-13.
- To find combinations close enough to satisfying Pythagorean Theorem, the paper provides a macro to generate an Excel spreadsheet of 68 combinations that one can sort by side/hypotenuse lengths, error, or slope (defined here as long leg divided by short leg). Just about all are feasible, and 11 have an (absolute) error of less than 0.1 LEGO units in the hypotenuse.

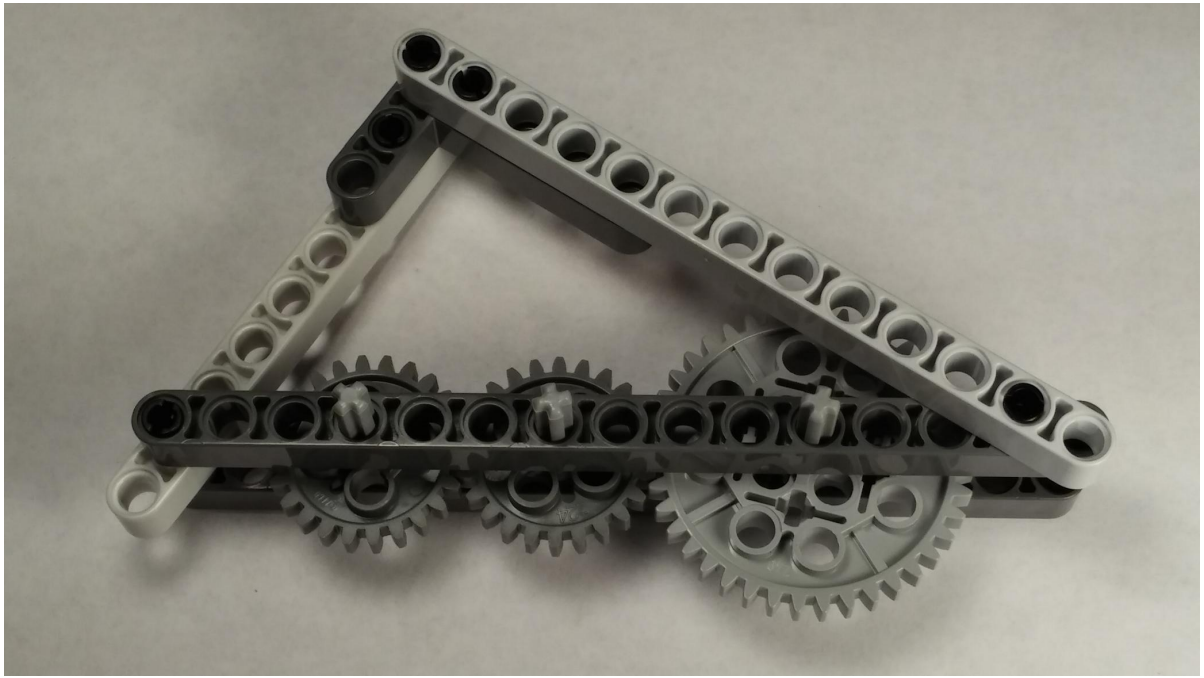
The VBA Macro

```
Sub Initialize()
Dim MAXLEG, MAXHYP, FRAC, HYPRND As Integer
MAXLEG = 14    'Maximum allowed leg length
MAXHYP = 14    'Maximum allowed hypotenuse length
MAXSLOPE = 5   'Maximum allowed slope (2nd leg divided by short leg)
FRAC = 1       'Fractions of Lego units allowed (1 for whole units only, 2 for halves, 4 for quarters, etc.)
HYPRND = 1     'Approximate hypotenuse by rounding to specified fraction (1 for whole units, 2 for halves, 4 for quarters,
etc.)
Cells.Clear
Cells(1, 1).Value = "Short Leg"       'A1
Cells(1, 2).Value = "2nd Leg"         'B1
Cells(1, 3).Value = "Hypotenuse"      'C1
Cells(1, 4).Value = "Approx Hyp"      'D1
Cells(1, 5).Value = "Error"           'E1
Cells(1, 6).Value = "Abs Err"         'F1
Cells(1, 7).Value = "Slope"           'G1
Dim shortstep, secondstep As Integer     'Counters for short, 2nd leg lengths
Dim row As Integer: row = 2              'Counter starting at 1st row after headings
For shortstep = 1 To MAXLEG * FRAC
   For secondstep = shortstep To MAXLEG * FRAC
      Cells(row, 1).Value = shortstep/FRAC                                    'A row
      Cells(row, 2).Value = secondstep/FRAC                                   'B row
      Cells(row, 3).Formula = "=SQRT(A" & row & "^2+B" & row & "^2)"          'C row
      Cells(row, 4).Formula = "=MROUND(C" & row & "," & 1/HYPRND & ")"        'D row
      Cells(row, 5).Formula = "=D" & row & "-C" & row                         'E row
      Cells(row, 6).Formula = "=ABS(E" & row & ")"                            'F row
      Cells(row, 7).Formula = "=B" & row & "/A" & row                         'G row
      If Cells(row, 4)<=MAXHYP And Cells(row, 7)<=MAXSLOPE Then row = row+1
   Next secondstep
Next shortstep
Rows(row).EntireRow.Delete     'Remove last row failing condition at loop end
End Sub
```

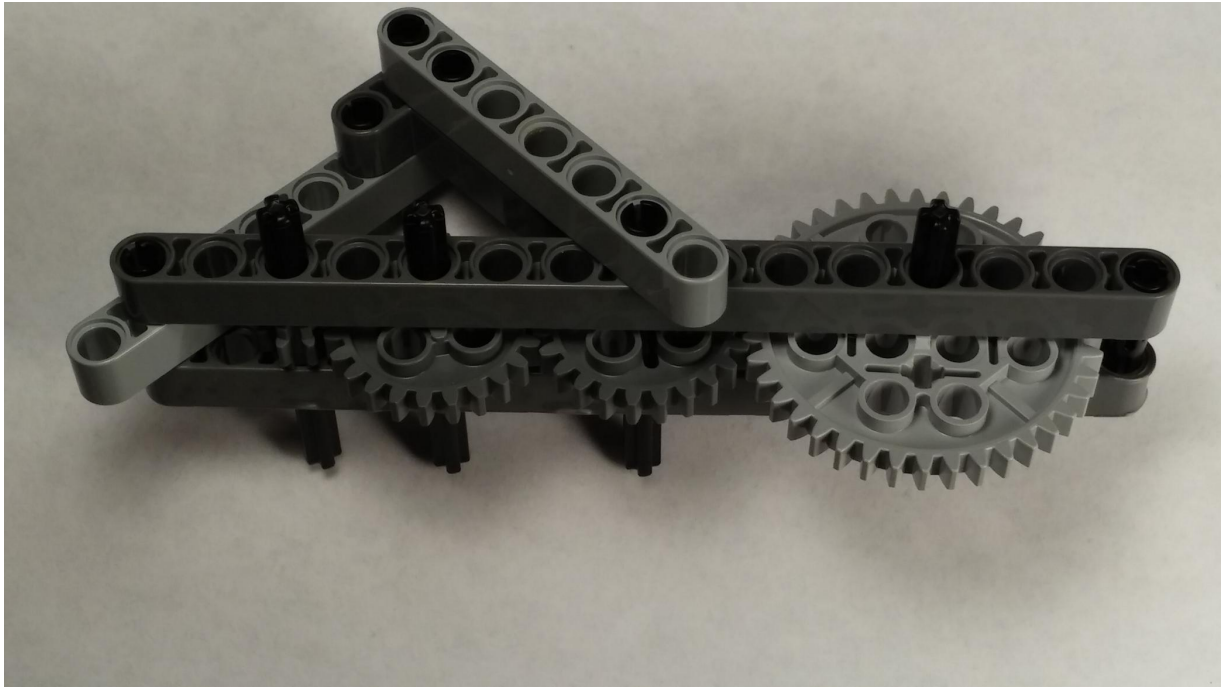
Near-Integral Triples with Least Absolute Error

Short Leg	2nd Leg	Approx. Hypotenuse	Error
7	11	13	-0.038
8	9	12	-0.042
4	8	9	0.056
4	7	8	-0.062
5	5	7	-0.071
5	13	14	0.072
5	11	12	-0.083
1	5	5	-0.099

Example: Approx. Triple with Least Absolute Error (7-11-13)

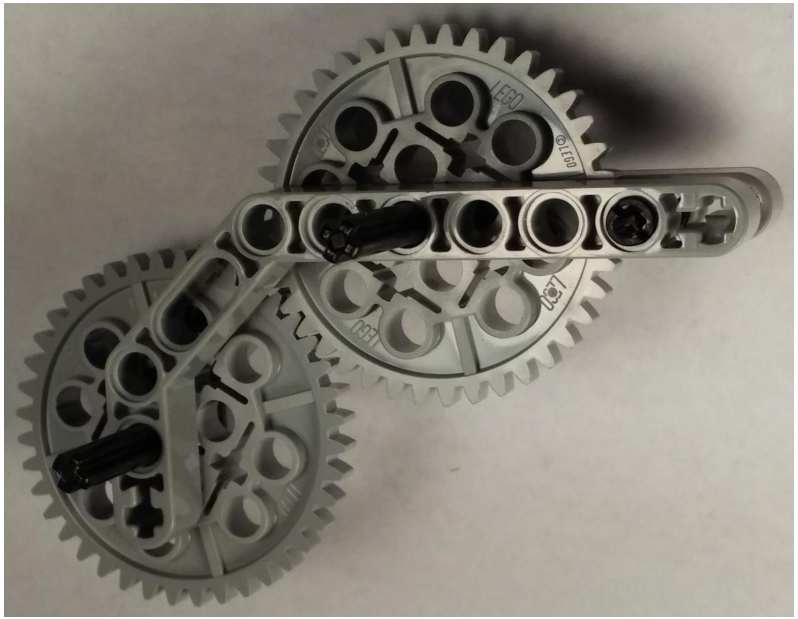


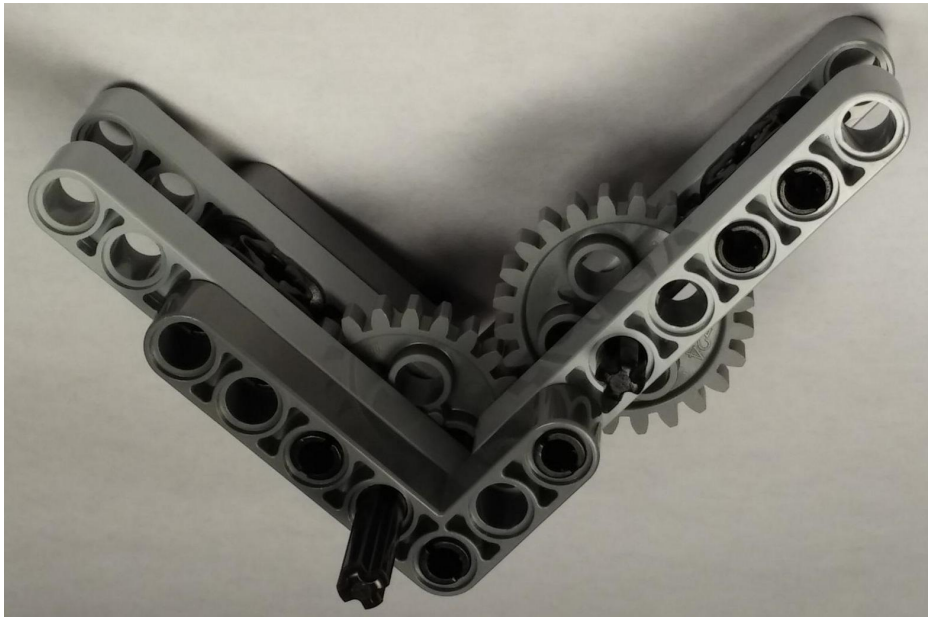
Example: Least Absolute Error for Slope 1 (5-5-7)



Example: Hypotenuse with Gears Only

Exact 3-4-5 and largest feasible absolute error (1-3-3)





Half-Unit Spacing

Various parts can be used to place pieces at half-unit spacing within the implicit underlying 3D LEGO grid:

Description	Part #	Quantity
Bush 1/2	32123	42
Nut 8-32 Keeps (black)		85
1 x 3 Liftarm Thin	6632	16
Triangle	2905	4
Cam	6575	4

Triangles with Half-Unit Sides

- Can modify macro to generate spreadsheet entries including half-unit sides.
- Can place gears along a half-unit side by using 16-Tooth with others at spacing of 1.5, 2.5, or 3.5, or new trick with double bevel gears at spacings of 1.5, 2, 2.5, 3, 3.5, or 4.5:

Description	Part #	Radius in LEGO Units	Quantity
12 Tooth Double Bevel	32270	0.75	4
20 Tooth Double Bevel	32269	1.75	8
36 Tooth Double Bevel	32498	2.25	6

Quarter-Unit Spacing

- Generally harder to achieve, but can at least pair a traditional gear with a double bevel gear along a hypotenuse at spacings of 1.25, 1.75, 2.25, 2.75, 3.25, 3.75, or 4.75.
- A further tweak of the Excel macro can generate approximate Pythagorean triples with such a hypotenuse.

Cataloging Gear Ratios and Spacings

- Another simple program with a double-loop
- Can introduce abstract data type, e.g.

```
Type gear
    name As String
    teeth As Integer
    radius As Single
End Type
```

- Or generate a CSV file of results using another programming language; could even use Scratch!

Conclusion

Other odd spacings may also be achievable by interposing metal pieces, bricks, or plates/tiles between liftarms, but we have shown a systematic approach to employ strong diagonal structures while using primarily pieces that are plentiful in the Botball kit and keeping liftarms, gear centers, etc. on a standard grid or half-unit grid.